

Hunting Dragons with Binary Search

The morning's news was nothing short of dismal. King Fredrick had only been awake for twenty minutes when he decided that today was going to be awful. It had started manageably enough, with his head butler informing him that his favorite crown was still being polished and was not available. However, that news was quickly followed by his chief knight, Sir Braver, informing King Fredrick that a dragon was now terrorizing his kingdom.

"Tell me what happened," commanded the king in a weary tone. He hated dragons.

"There is a dragon," declared Sir Braver.

"You have already told me that part. Now tell me what has happened. Has anyone seen this dragon? Has it attacked anyone?" prompted the king.

"Yes, sir. We have received six confirmed sightings. Farmer McDonald has lost his prize cow."

"The one that won last year's competition?" inquired the king. If he recalled correctly, it was a sizable cow and would be a perfect target for the dragon.

"Yes, sir," answered the knight.

The king sighed. The story sounded like a textbook dragon attack. Soon the dragon would eat half the cows in the kingdom. "Well, you had better take the dragon hunter and get rid of the dragon."

"But sir, no one knows where the dragon is. How shall we hunt it?" asked the knight.

"Really?" the king asked, surprised that his head knight did not know how to hunt a dragon.

Sir Braver remained silent.

"Find the last farm that it attacked," said the king. "Dragons take long naps after each meal, so you have time to catch it there."

"But sir, how will we know where that is?" asked the knight.

"Have you studied dragon attacks at all?" the king asked. "There is an order to these attacks. Dragons are smart. They eat the biggest cow in the area, then the second biggest, then the third, and so forth. They keep eating smaller and smaller cows, until the only remaining cows are too small to be worth the bother. Then the dragon goes on to the next kingdom."

"But sir..." started the knight.

"Use the rankings from last year's cow competition!" cried the king, losing his patience. "They have a sorted list of the largest cows!"

"Ah. Of course, sir. We shall work down the list until we find the dragon!" confirmed the knight.

The king nodded, relieved that the knight was finally thinking on his own. Sir Braver and the dragon hunter would visit each of the farms on the list, in order, and find the dragon. Even as he tried to comfort himself with this thought, warning bells went off in the king's head.

"Not down the list," the king admonished. "Do you know anything about searching?"

The knight looked confused. He had been certain that he was on the correct track. "But sir, I don't understand" he said.

"How long does it take you to travel between farms?" asked the king.

"A few hours, sir."

"How many farms are on the list?"

"Maybe a hundred."

"That means that it could take you hundreds of hours. By that time, the dragon will have eaten more cows. It might even be gone by then."

"But, sir..."

"Use binary search," commanded the king.

"Binary search, sir?" asked Sir Braver, regretting having fallen asleep in his 'Algorithms for Knights' class.

"Start in the middle of the list and check whether the cow is still there. If it is still there: rip the list in half, throw away the bottom half, and repeat the process on the top half of the list. If the cow is gone: rip the list in half, throw away the top half, and repeat the process on the bottom half of the list. Each time you visit a farm, rip the list in half and concentrate your search on only one of the two halves."

"But sir, that sounds complex," protested Sir Braver.

"It is not," argued the king. "It is simple. If the cow in the middle of the list is still at the farm, then so are all the cows below it on the list. That is how dragons eat. It is not going to eat a smaller cow first. That would be absurd."

"On the other hand, if the cow is missing, then so are all the cows above it on the list. Dragons do not skip around lists randomly. There is an order to these things."

"What if the dragon moves while we are searching?" asked the knight.

Finally, a reasonable question. "Your search effectively tracks two bounds: the smallest cow that you know has been eaten and the largest cow that you know has not been eaten. If you get to a point where those bounds are next to each other on the list, and you do not see a dragon, then proceed to the largest uneaten cow and wait there."

"But sir, how is that any faster than just scanning down the list?" asked the knight.

The king sighed, again. His confidence in the knight's ability to handle this task was eroding rapidly. "If the dragon has attacked 45 farms, how long will it take you to scan those farms? 90 hours? But in the case of binary search: after the first farm you will have ruled out 50 farms on the list -- one way or another. After the second farm, you will have ruled out another 25. Each time, you are cutting the problem in half. In fact, you can find the dragon after checking only 7 farms."

The knight nodded yet looked unconvinced. "But sir, are you sure?"

The king screamed. "Of course, I am sure! I am the king! And, further, every idiot knows that binary search is a logarithmic time algorithm while scanning down a list is a linear time algorithm. NOW GO AND GET THAT DRAGON!"

For the first time in his long career as a knight, Sir Braver ran away. He told himself that he was running in order to carry out the king's orders as quickly as possible. But in truth, the king looked really mad.

As he watched his top knight clank noisily down the hall, the king decided that it was indeed time to go back to bed.

